

Integrable system with peakon, weak kink, and kink-peakon interactional solutions

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Abstract:

In this presentation, we present an integrable system with both quadratic and cubic nonlinearity: $m_t = bu_x + \frac{1}{2}k_1 [m(u^2 - u_x^2)]_x + \frac{1}{2}k_2(2mu_x + m_xu)$, $m = u - u_{xx}$, where b , k_1 and k_2 are arbitrary constants. This model is kind of a cubic generalization of the Camassa-Holm (CH) equation: $m_t + m_xu + 2mu_x = 0$. The equation is shown integrable with its Lax pair, bi-Hamiltonian structure, and infinitely many conservation laws. In the case of $b = 0$, the peaked soliton (peakon) and multi-peakon solutions are studied. In particular, the two-peakon dynamical system is explicitly presented and their collisions are investigated in details. In the case of $b \neq 0$ and $k_2 = 0$, the weak kink and kink-peakon interactional solutions are found. Significant difference from the CH equation is analyzed through a comparison. Finally, we also show all possible smooth one-soliton solutions for the system.

References:

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